Math 60   9.7 Functions Involving Radicals

Objectives
1) Evaluate Functions Involving Radicals
2) Find the domain of a function involving a radical.
3) Graph functions involving square roots
4) Graph functions involving cube roots.

① $f(x) = \sqrt{x+2}$

$g(x) = \sqrt[3]{3x+1}$

a) Find $f(14)$
b) Find $f(6)$
c) Find $g(21)$.
d) Find $f(-3)$

a) $f(14) = \sqrt{14+2}$ replace x by 14

$= \sqrt{16}$ evaluate inside out

$= \boxed{4}$ take square root

b) $f(6) = \sqrt{6+2}$ replace x by 6

$= \sqrt{8}$ evaluate inside out

not a perfect square $\Rightarrow$ simplify instead, if possible

$= \sqrt{4 \cdot 2}$

$= \boxed{2\sqrt{2}}$

c) $g(21) = \sqrt[3]{3(21)+1}$ replace x by 21

$= \sqrt[3]{63+1}$ evaluate from inside out

$= \sqrt[3]{64}$

$= \boxed{4}$ take cube root

d) $f(-3) = \sqrt{-3+2}$ replace x by -3

$= \sqrt{-1}$ evaluate inside out

$=$ $\boxed{\text{not a real \#}}$

Math 60 9.7

When graphing, the y-coordinates must be real numbers.
The domain of a function is the set of all x-values which have real y-coordinates.

To find the domain of $f(x) = \sqrt[n]{\text{stuff}}$

<u>Step 1:</u> If n is odd, $\sqrt[n]{\text{neg}}$ are real, so all x are permitted.
The domain is all real numbers.

<u>Step 2:</u> If n is even, $\sqrt[n]{\text{neg}}$ is not real, so $\sqrt[n]{\text{stuff}}$ must have stuff ≥ 0.

(2) Find the domain. Write in set notation and interval notation.

a) $f(x) = \sqrt{x-2}$

index $n = 2$ is even

stuff ≥ 0 — radicand must be positive or zero

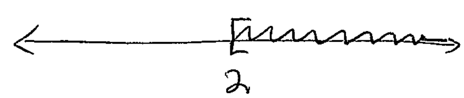
$x - 2 \geq 0$ — Solve inequality by isolating x.

$+2 \quad +2$

$x \geq 2$

Set notation $\boxed{\{x \mid x \geq 2\}}$

graph



interval $\boxed{[2, \infty)}$

b) $g(x) = \sqrt[3]{7x-2}$

index $n = 3$ is odd

domain is all real numbers

set notation $\{x \mid x \text{ is a real number}\}$

interval $(-\infty, \infty)$

Math 60 9.7

c) $h(z) = \sqrt[4]{3-2z}$

$n=4$ is even, so radicand must be positive or zero.

$$\begin{aligned} 3 - 2z &\geq 0 \\ -3 \quad & \quad -3 \end{aligned}$$

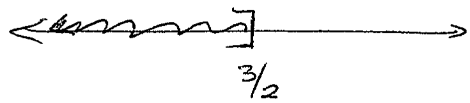
solve inequality by isolating x

$$\frac{-2z}{-2} \geq \frac{-3}{-2}$$

$$z \leq \frac{3}{2}$$

* switch direction of inequality when divide by a negative

set notation $\boxed{\{z \mid z \leq \frac{3}{2}\}}$

graph 

interval notation $\boxed{(-\infty, \frac{3}{2}]}$

Graph the function by plotting points.

3) $f(x) = \sqrt{x}$

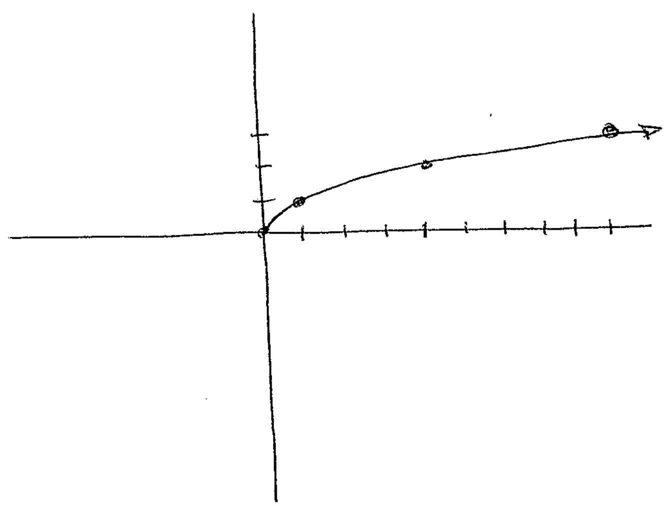
x	y
0	
1	
4	
9	

⇑

choose x-values which are perfect squares

Notice: The square root graph is half a parabola, turned on its side.

x	$y=\sqrt{x}$	
0	$\sqrt{0} = 0$	(0, 0)
1	$\sqrt{1} = 1$	(1, 1)
4	$\sqrt{4} = 2$	(4, 2)
9	$\sqrt{9} = 3$	(9, 3)



Math 60 9.7

(4) $g(x) = \sqrt{x-2}$

Use domain to help choose x-values
$x-2 \geq 0$
$x \geq 2$ ← 2 is smallest x.

x	y
2	
3	
6	
11	

$x-2=0$
$x=2$

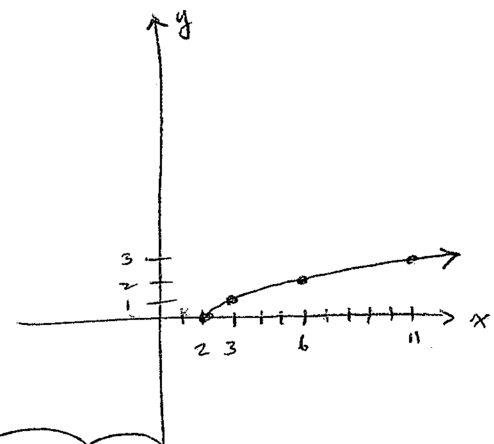
$x-2=1$
$x=3$

$x-2=4$
$x=6$

$x-2=9$
$x=11$

choose x-coordinates so that the radicand equals the perfect square numbers we used before.

x	y	
2	$\sqrt{2-2} = \sqrt{0} = 0$	$(2,0)$
3	$\sqrt{3-2} = \sqrt{1} = 1$	$(3,1)$
6	$\sqrt{6-2} = \sqrt{4} = 2$	$(6,2)$
11	$\sqrt{11-2} = \sqrt{9} = 3$	$(11,3)$



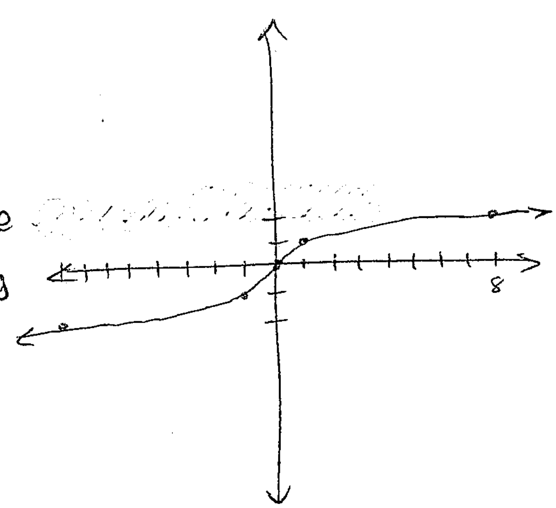
(5) $h(x) = \sqrt[3]{x}$

domain is all real numbers
Need positive and negative x-values

x	y	
0	$\sqrt[3]{0} = 0$	$(0,0)$
1	$\sqrt[3]{1} = 1$	$(1,1)$
8	$\sqrt[3]{8} = 2$	$(8,2)$
27	$\sqrt[3]{27} = 3$	$(27,3)$
-1	$\sqrt[3]{-1} = -1$	$(-1,-1)$
-8	$\sqrt[3]{-8} = -2$	$(-8,-2)$
-27	$\sqrt[3]{-27} = -3$	$(-27,-3)$

choose x-coordinates which are perfect cubes

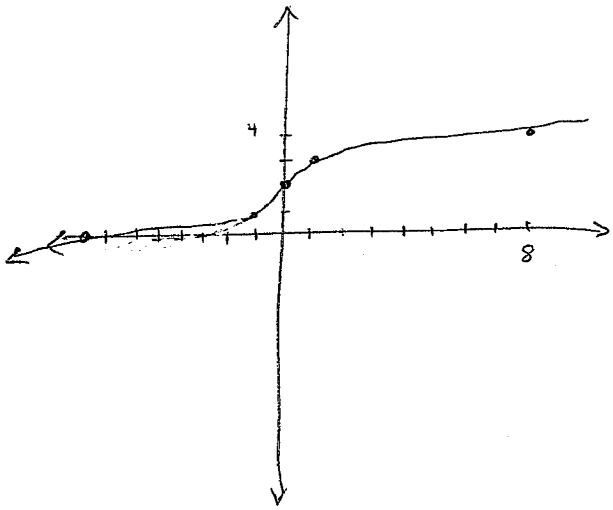
odd index ⇒ use negative x-coordinates also



Math 60 9.7

(6) $f(x) = \sqrt[3]{x} + 2$ domain is all real numbers. Need (+) and (−) x-values.

x	y	
0	$\sqrt[3]{0}+2=2$	(0, 2)
1	$\sqrt[3]{1}+2=3$	(1, 3)
8	$\sqrt[3]{8}+2=4$	(8, 4)
27	$\sqrt[3]{27}+2=5$	(27, 5)
−1	$\sqrt[3]{-1}+2=1$	(−1, 1)
−8	$\sqrt[3]{-8}+2=0$	(−8, 0)
−27	$\sqrt[3]{-27}+2=-1$	(−27, −1)



Trickier HW questions:

(7) Find the domain of $f(x) = \sqrt{\frac{3}{x-4}}$

Recall that the domain of $\frac{3}{x-4}$ is all real numbers

except where denominator $x-4=0$

$x \neq 4$.

Here: We want $\frac{3}{x-4} \geq 0$ We'll learn how to solve this question in chapter 10!

Notice that 3 in the numerator is always positive, so we need the denominator to be positive

$x-4>0$

$x>4$

set notation $\boxed{\{x \mid x>4\}}$

graph

4

interval notation $\boxed{(4, \infty)}$